臺灣綜合大學系統105學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	A07/D01
		科目碼	A0702
※本項考試依簡章規定各考科均「不可以」使用計算機		本試題共計	1 頁

1. (16%) Consider the following two subspaces of $\mathbb{R}^5$:

$$W_1 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 \mid a_1 - a_2 - 2a_4 = 0\}$$
$$W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 \mid a_2 = a_3 \text{ and } a_1 + a_5 = 0\}.$$

(a) Find bases for W_1 and W_2.
(b) Evaluate the dimension of $W_1 + W_2$.

2. (32%) Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$.

(a) Determine all the eigenvalues of A and find the corresponding eigenspaces.
(b) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.
(c) Is A invertible?
(d) Solve the system $Ax = b$.

3. (24%) Let $T\colon P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be the linear transformation defined by

$$T(f(x)) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix}.$$

(a) Find bases for both $N(T)$ and $R(T)$.
(b) Evaluate the nullity and rank of T.
(c) Is T one-to-one?

4. (8%) Suppose that λ, μ are distinct eigenvalues of A with corresponding eigenspaces E_λ, E_μ. Show that $E_\lambda \cap E_\mu = \{0\}$.

5. (10%) Show that similar matrices have the same characteristic polynomial.

6. (10%) Let $T\colon V \to W$ be a linear transformation and let $\{w_1, w_2, \ldots, w_k\}$ be a linearly independent subset of $R(T)$. Show that if $S = \{v_1, v_2, \ldots, v_k\}$ is chosen so that $T(v_i) = w_i$ for $i = 1, 2, \ldots, k$, then S is linearly independent.