臺灣綜合大學系統105學年度學士班轉學生聯合招生考試試題

科目名稱	微積分C	類組代碼	E00
		科目碼	E0013
※本項考試依簡章規定各考科均「不可以」使用計算機		本試題共計	2 頁

一、 填充題(不需計算過程) 請於答案卷上作答，否則不予計分

1. (12 points) Find the following limits:

   (a) (6 points) For $a, b, c > 0$, $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}} =$ _____.

   (b) (6 points) Denote $\prod_{k=1}^{n} A_k = A_1 A_2 \cdots A_n$. Evaluate $\lim_{n\to\infty}\prod_{k=1}^{n}\left(1+\frac{k}{n^2}\right) =$ _____.

2. (5 points) Suppose that the function $f(x) = \begin{cases} \frac{ax+b}{\sqrt{3x+1}-\sqrt{x+3}} & , x \neq 1 \\ 4 & , x = 1 \end{cases}$ is continuous at $x = 1$. Then $(a, b) =$ _____.

3. (5 points) Define $f(x) = \tan^3 x$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$ and let f^{-1} be the inverse function of f. Find $\left(f^{-1}\right)'(3\sqrt{3}) =$ _____.

4. (12 points) Compute the integrals

   (a) (6 points) $\int \frac{\ln x}{(1-x)^2}\,dx =$ _____

   (b) (6 points) $\int_1^{\infty} \frac{1}{e^{x+1}+e^{3-x}}\,dx =$ _____.

5. (6 points) Let h be a differentiable function of x and y and let $f(r, s) = h(rs, r+s)$. Assume that $\frac{\partial h}{\partial x}(6,5) = 1$ and $\frac{\partial h}{\partial y}(6,5) = 2$. Find $\frac{\partial f}{\partial s}(2,3) =$ _____.

二、 計算題(無計算過程不給分)

6. (10 points) Find the general solution of the differential equation

$$(x^2+1)y' + 2xy = 4x^2$$

7. (10 points) Consider the series $\sum_{k=10}^{\infty} \frac{1}{k \ln k\left[\ln(\ln k)\right]^p}$. Determine all values of p such that the series converges.

8. (20 points) Let $f(x) = \frac{1}{\sqrt[3]{1+x^4}}$.

   (a) (10 points) Find the Taylor series of $f(x)$ at $x = 0$. (*Need to write down the general form.*)

   (b) (10 points) Find the interval of convergence of the Taylor series in Problem(a).

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9. (10 points) Let $f(x,y) = x^2 + y^2 - 12x + 16y$. Find the maximum of f on the set $\{(x,y) \mid x^2 + y^2 \leq 25\}$.

10. (10 points) Let E be the solid cone bounded below by $z = \sqrt{x^2 + y^2}$ and above by $z = 2$. Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Evaluate

$$\iiint_E \text{div}\mathbf{F}\, dV$$