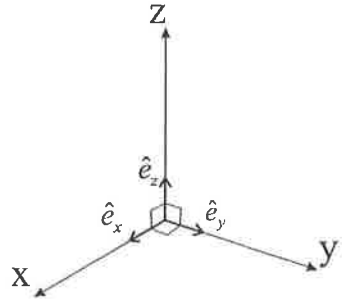
臺灣綜合大學系統 105 學年度學士班轉學生聯合招生考試試題

科目名稱	力學	類組代碼	D13
		科目碼	D1392
※本項考試依簡章規定各考科均「不可以」使用計算機		本科試題共計 9 頁	

本試卷共 12 題，每題均為單選題。答錯均不倒扣。

1. [5%] A vector $\vec{r}$ is denoted as $\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, where $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are Cartesian unit vectors. Given three vectors $\vec{A} = \hat{e}_x + \hat{e}_y, \vec{B} = \hat{e}_x - \hat{e}_y, \vec{C} = \hat{e}_z$. Find $\vec{A}\times(\vec{B}\times\vec{C})$.

(A) $\hat{e}_y$
(B) 0
(C) $\hat{e}_x + \hat{e}_y$
(D) $-\hat{e}_x$



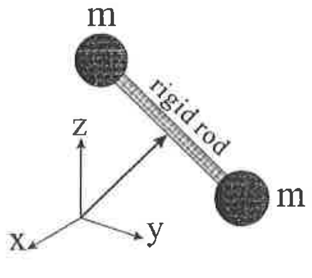
2. [5%] Consider a particle with mass m subjected to the one dimensional potential energy function $V(x)$. If $x = 0$ is the position of stable equilibrium, the particle oscillates harmonically about the equilibrium position with what frequency?

(A) $\sqrt{V_0''/(2m)}$
(B) $\sqrt{V_0''/m}$
(C) $\sqrt{V_0'/m}$
(D) $\sqrt{V_0'/(2m)}$,

where $V_0'' = \left.\dfrac{d^2V}{dx^2}\right|_{x=0}$, and $V_0' = \left.\dfrac{dV}{dx}\right|_{x=0}$.

3. [5%] Find the degrees of freedom of the system: two masses connected by a rigid rod.

(A) 2
(B) 3
(C) 4
(D) 5



4. [5%] Find the equation of motion generated by the Lagrangian: $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, where m is mass, k is a constant, and x is the generalized coordinate.

(A) $m\ddot{x} + kx = 0$
(B) $m\ddot{x} - kx = 0$
(C) $m\ddot{x} + kx^2 = 0$
(D) $m\dot{x}^2 + kx^2 = 0$.

背面有題，請繼續作答。

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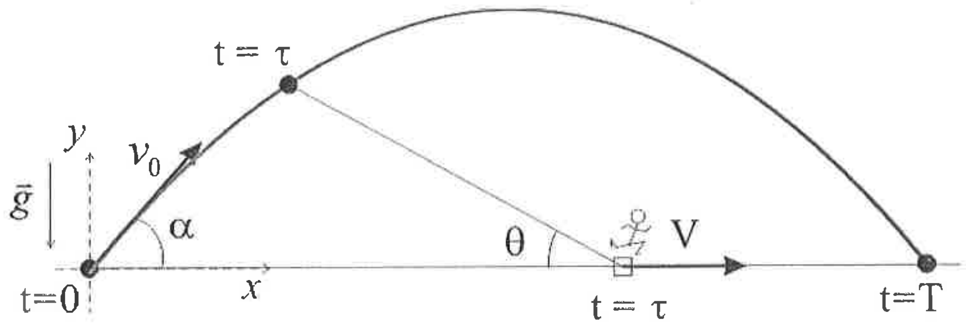
5. [10%] In a baseball game, consider the motion of a ball in a uniform gravitation field (the acceleration of gravity $\vec{g}$ is a constant) without air resistance. The ball is launched at origin of the coordinate with the initial speed v_0 making an angle α with the horizontal axis, as shown in the figure. After time τ, the fielder starts to move with velocity V. If the fielder's velocity is correct, he will catch the ball at $x = R$ and $t = T$, where R is the position where the ball hits the ground and T is the flight time of the ball. The fielder then makes an angle θ that is the line-of-sight elevation to the ball's instantaneous position. As the fielder's velocity is correct, it can be shown that $\tan\theta$ is proportional to t, namely, $\tan\theta = \text{constant} \times t$. Find the constant.



(A) $\dfrac{g}{(v_0 \sin\alpha + V)}$

(B) $\dfrac{2g}{(v_0 \cos\alpha - V)}$

(C) $\dfrac{g}{2(v_0 \cos\alpha - V)}$

(D) $\dfrac{g}{2(v_0 \sin\alpha - V)}$

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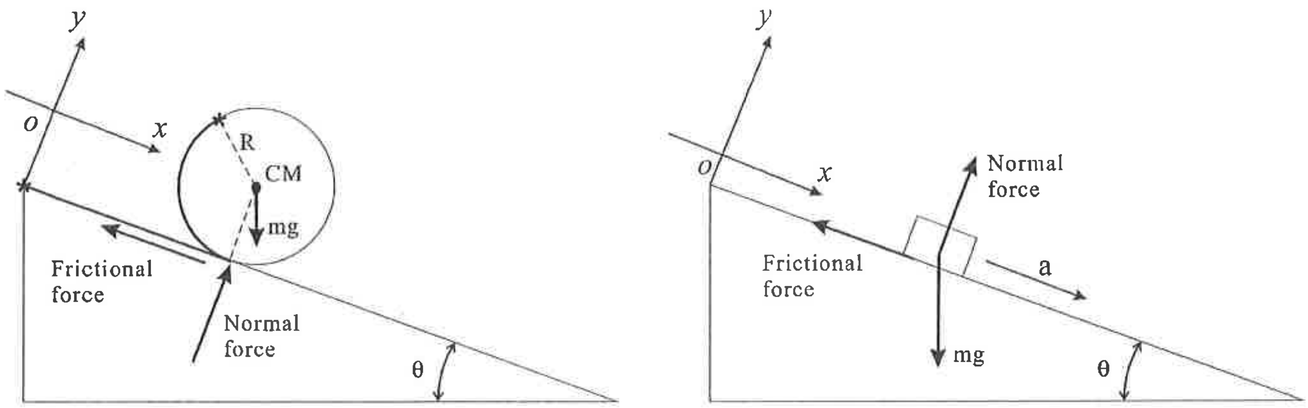
6. [10%] Consider a solid uniform sphere (radius R and mass m) rolling down an inclined plane at angle θ to horizontal, and the contact is very rough so that no slipping can occur, as shown in the following figure.



Compare to the sliding motion of mass m on a rough plane at angle θ, where the kinetic friction is μ_k. At what angle θ the two masses will have the same linear acceleration of the center of mass? (the moment of inertia of the uniform solid sphere with radius R is $I=\frac{2}{5}mR^2$, where the rotation axis passes through the center).

(A) $\tan^{-1}(\mu_k)$

(B) $\tan^{-1}\left(\frac{5}{2}\mu_k\right)$

(C) $\tan^{-1}\left(\frac{3}{2}\mu_k\right)$

(D) $\tan^{-1}\left(\frac{7}{2}\mu_k\right)$

 背面有題，請繼續作答。

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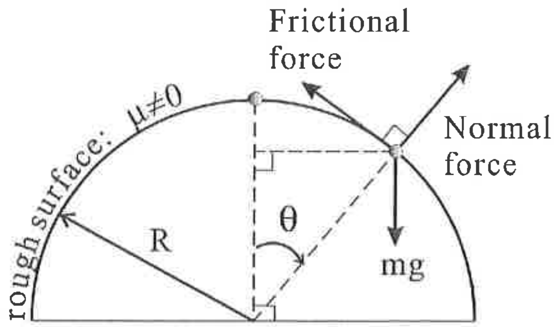
7. [10%] A small mass m rests on top of a fixed sphere of radius R. The coefficient of kinetic friction of the sphere surface is denoted as μ, as shown in the following figure.



The mass is slightly disturbed, and it slides down along the surface of the rough sphere. Find the equation of motion of the small mass.

(A) $\ddot{\theta}+\mu\dot{\theta}^2=-\dfrac{g}{R}(\sin\theta-\mu\cos\theta)$

(B) $\ddot{\theta}-\mu\dot{\theta}^2=\dfrac{g}{R}(\sin\theta-\mu\cos\theta)$

(C) $\ddot{\theta}-\mu\dot{\theta}^2=\dfrac{g}{R}(\sin\theta+\mu\cos\theta)$

(D) $\ddot{\theta}+\mu\dot{\theta}^2=\dfrac{g}{R}(\sin\theta-\mu\cos\theta)$

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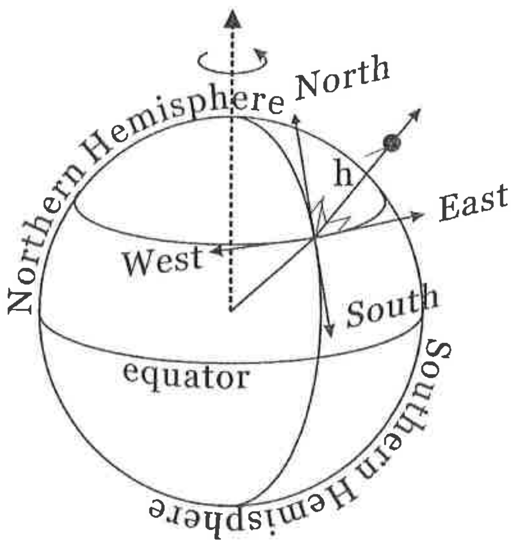
8. [10%] Consider the Earth's rotation effect. At northern hemisphere, suppose a body is dropped from rest at a height h above the earth ground, as shown in the figure below.



Earth turns to the east. What direction does the body dominantly drift when it hits the ground?

(A) Northward drift
(B) Westward drift
(C) Eastward drift
(D) Southward drift

背面有題，請繼續作答。

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9. [10%] In 1799 and during 1920, Laplace, Runge and Lenz have shown that there exists a vector $\vec{A}$, called L-R-L vector, that is a constant of motion in the central force field $\vec{F}=-k/r^2$. This L-R-L vector plays an important role in the explaining the accidental degeneracy of the spectrum of hydrogen atom in quantum physics. Which one is the L-R-L vector, i.e. which vector is a constant of motion in the central force $\vec{F}=-k/r^2$.

(A) $\vec{A}=-\dfrac{\vec{r}}{r}+\dfrac{\vec{v}\times\vec{L}}{k}$

(B) $\vec{A}=-\dfrac{\vec{r}}{r}-\dfrac{\vec{p}\times\vec{L}}{k}$

(C) $\vec{A}=-\dfrac{\vec{r}}{2r}+\dfrac{\vec{v}\times\vec{L}}{k}$

(D) $\vec{A}=-\dfrac{\vec{r}}{r}+\dfrac{\vec{p}\times\vec{L}}{k}$,

where $\vec{r}$ is the position vector, $r=|\vec{r}|$, $\vec{v}$ is the particle's velocity, $\vec{p}$ is the momentum of the particle, and $\vec{L}$ is the orbital angular momentum.

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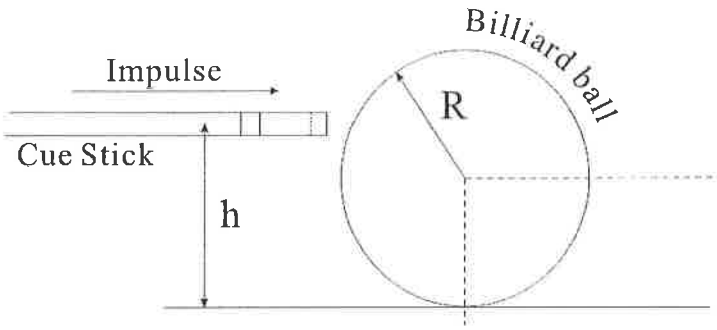
10. [10%] We use a cue stick to impart an impulse to a billiard ball with radius R, as shown in the following figure. Assume that the ball is a solid and uniform sphere. At what height h, the ball will roll without slipping, which is called the rolling shot.



(A) $\frac{3}{2}R$

(B) $\frac{5}{3}R$

(C) $\frac{7}{5}R$

(D) $\frac{2}{5}R$

背面有題，請繼續作答。

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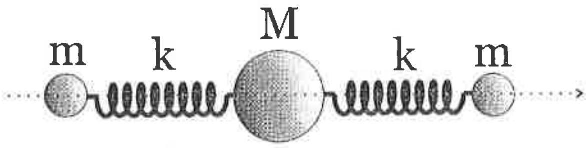
11. [10%] The general vibrating motion of coupled harmonic oscillators can be obtained by using linear combinations of system's normal modes. Consider the linear motion of a triatomic molecule, which all particles lie in a straight line, and they are connected by the same springs with spring constant k, as shown in the following figure.



What is the highest normal mode frequency?

(A) $\sqrt{\dfrac{k}{M}+\dfrac{k}{2m}}$

(B) $\sqrt{\dfrac{k}{2m}+\dfrac{k}{M+m}}$

(C) $\sqrt{\dfrac{2k}{m}+\dfrac{k}{M}}$

(D) $\sqrt{\dfrac{k}{m}+\dfrac{2k}{M}}$.

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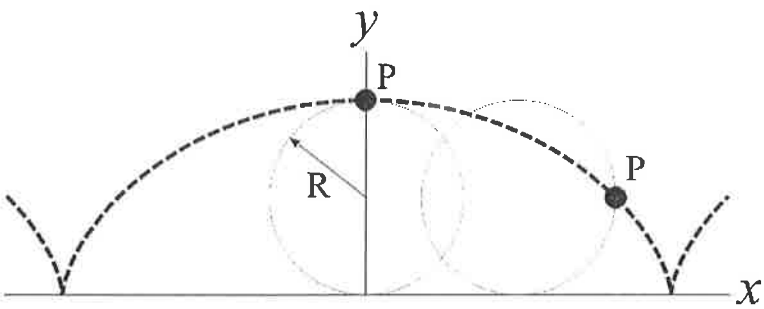
12. [10%] In 1696, John Bernoulli showed that in a uniform gravitation field the elapsed time for a bead sliding along the frictionless cycloidal curve in the vertical x, y plane is minimum. The cycloidal curve can also be obtained by the path of a particle P on a rolling wheel, as shown in the figure (dashed line). Assume that the wheel has a radius R and constant angular velocity ω.



Find the equation of the cycloid path generated by the path of the particle on the rolling wheel.

(A) $y = R\cos^{-1}\left(\frac{x}{2R}+1\right)+\sqrt{2Rx+x^2}$

(B) $x = R\cos^{-1}\left(\frac{y}{R}-1\right)+\sqrt{2Ry-y^2}$

(C) $x = R\cos^{-1}\left(\frac{y}{R}-1\right)-\sqrt{2Ry+y^2}$

(D) $y = R\cos^{-1}\left(\frac{x}{2R}-1\right)+\sqrt{2Rx-x^2}$