臺灣綜合大學系統 105 學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	D25
		科目碼	D2592
※本項考試依簡章規定各考科均「不可以」使用計算機		本試題共計	1 頁

1. [10 points] Let A be an $n \times n$ real matrix and $A^\top = -A$. Show that $\det(A) = 0$ if n is odd.

2. [10 points] Let A and B be two $n \times n$ nonsingular matrices. Show that $A^{-1}(A^{-1} + B^{-1})^{-1} = (A+B)^{-1}B$.

3. [30 points] Suppose $V = R^{2\times 2}$. Let V_1 and V_2 be two subspaces of V defined by

$$V_1 = \{\begin{bmatrix} a & a+b \\ 0 & -b \end{bmatrix} | a, b \in \mathbb{R}\} \text{ and } V_2 = \{\begin{bmatrix} a & b \\ 2a-b & -a \end{bmatrix} | a, b \in \mathbb{R}\}.$$

   (a) [10 points] Determine the dimensions of V_1 and V_2.

   (b) [10 points] Determine the dimension of $V_1 + V_2$.

   (c) [10 points] Determine the dimension of $V_1 \cap V_2$.

4. [30 points] Let P_2 be a vector space consisting of all polynomials of degree at most two and let $T : P_2 \to P_2$ be the linear transformation satisfying $T(p(x)) = p(x-1)$ for any polynomial $p(x)$ in P_2.

   (a) [10 points] Find the matrix A representing T with respect to the ordered basis $\{1, x, x^2\}$.

   (b) [10 points] Find the matrix B representing T with respect to the ordered basis $\{1, x+1, x^2+1\}$.

   (c) [10 points] Find an invertible matrix S such that $AS = SB$ and the first column of S is $[1, 0, 0]^\top$.

5. [10 points] Let $A = \begin{bmatrix} 5 & 0 & 1 & 1 \\ 0 & 5 & 4 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. Find a matrix P, where the first column of P is $[1, 0, 0, 0]^\top$, that diagonalizes A and determine $P^{-1}AP$.

6. [10 points] Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Compute the value of e^A.