科目名稱	微積分 C	類組代碼	
		科目碼	E0013

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 **1** 頁

*** Show All Your Work. No Electronic Devices Allowed ***

1 (10%). Find the limit

$$\lim_{x \to 0^+} \left[\frac{1}{\ln(1+x)} - \frac{1}{x} \right].$$

2 (10%). Find all possible relative extrema and saddle points of the function

$$f(x, y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1.$$

3 (10%). Find the arc length of the polar curve:

$$r = 7^\theta, \quad 0 \le \theta \le 2\pi.$$

4 (10%). Compute the integral $\int_0^1 \sin^{-1}(x)dx$.

5 (10%). Let $J(x)$ be a function satisfying the differential equation $xJ''(x) + J'(x) + xJ(x) = 0$ for all values of x and $J(0) = 1$. Find $J''(0)$.

6 (10%). Determine whether the following series is convergent or divergent.

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left| \frac{16k^2}{n^3} - \frac{16k}{n^2} + \frac{3}{n} \right|$$

If it is convergent, find its sum. Otherwise, give a reason for your answer.

7 (10%). Let **u** and **v** be the unit normal vectors of the tangent planes of the surfaces $S_1 : x^2 + y^2 + z^2 + 2x - 4y - 4z = 12$ and $S_2 : 4x^2 + y^2 + 16z^2 = 24$ at the common point $(1, -2, 1)$. Compute the inner product $\mathbf{u} \cdot \mathbf{v}$.

8 (10%). Evaluate the integral $\iint_S xydS$, where the surface $S = \{(x, y, z) \mid x^2 + y^2 = 1, x \ge 0, y \ge 0, 0 \le z \le 1\}$.

9 (10%). Compute the integral

$$\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} \sqrt{x^2 + y^2}dydx.$$

10 (10%). Let R be the region bounded by the graph of $xy = 1$, $xy = 4$, $x = 2$, and $x = 3$. Compute the integral

$$\iint_R \frac{e^{-xy}}{1 + x^2}dA.$$