

臺灣綜合大學系統 106 學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	D25
		科目碼	D2592

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 **3** 頁

- There are 9 problems with 100 points in this test.
- Please show all your work for partial credits.

1. (15 pts) Let $T : R^3 \rightarrow R^3$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 2 & 3 & 8 \end{pmatrix}. \text{ Determine whether } T \text{ has an inverse. If so, find } T^{-1} \left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \right).$$

2. (15 pts) Calculate eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

Is A diagonalizable? If yes, find an invertible matrix P such that $P^{-1}AP$ is diagonal.

3. (10 pts) Find all solutions (if any) to the linear system:

$$\begin{cases} x_1 + 2x_2 - 3x_3 + 4x_4 = 2 \\ 2x_1 + 5x_2 - 2x_3 + x_4 = 1 \\ 5x_1 + 12x_2 - 7x_3 + 6x_4 = 4 \end{cases}$$

4. (10 pts) Prove that $S = \{(x, y, z) \mid 2x - 4y + 6z = 0\}$ is a subspace of R^3 . Find an orthonormal basis for S .

背面有題，請繼續作答。

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<p>5. (10 pts) Find a basis for the row space of the matrix</p> $\begin{pmatrix} 1 & 3 & 2 \\ 1 & 4 & 1 \\ 2 & 5 & 5 \end{pmatrix}.$			
<p>6. (10 pts) Determine whether the following statements are true or false. You must <u>explain</u> your answers for credits.</p> <p>(a) (5 pts) Let A be a square matrix. Suppose that $A^n = 0$ for some positive integer n. Then, A is singular.</p> <p>(b) (5 pts) Let $\{v_1, v_2, v_3\}$ be a basis for a vector space V. Then, the set</p> $\{v_1 - 2v_2 + 3v_3, 3v_2 - 4v_3, 2v_3\}$ <p>is also a basis for V.</p>			
<p>7. (10 pts) Let A be the following symmetric matrix:</p> $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$ <p>(a) (5 pts) Find an orthogonal matrix Q such that $Q^T A Q = D$ is a diagonal matrix. What is the diagonal matrix D?</p> <p>(b) (5 pts) Explain how to use this result to find the eigenvalues and eigenvectors of the matrix A^{10}?</p>			

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8. (10 pts) Let $T_1 : P_1 \rightarrow P_2$ be the linear transformation defined by

$$T_1(p(x)) = (2x + 1)p(x), \quad \forall p(x) \in P_1,$$

and let $T_2 : P_2 \rightarrow P_2$ be the linear transformation defined by

$$T_2(p(x)) = p(2x - 1), \quad \forall p(x) \in P_2.$$

Let us choose $B = \{1, x\}$ and $B' = \{1, x, x^2\}$ to be the basis for P_1 and P_2 , respectively. Find the matrix representation M for the transformation $(T_2 \circ T_1)$. What is the rank of the matrix M ?

9. (10 pts) Find all (nonequivalent) Jordan matrices with characteristic polynomial $p(\lambda) = (\lambda - 5)^4$. Furthermore, find the minimum polynomial for each of the Jordan matrices.