臺灣綜合大學系統 107 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	共同考科
		科目碼	E0011

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 2 頁

題號標示清楚，寫出計算過程否則不予計分，答案儘可能化簡。

1. (10 Points) Evaluate:

   (a) (5 Points) $\lim\limits_{n\to\infty} \dfrac{n}{n+1}$

   (b) (5 Points) $\lim\limits_{x\to 0} e^{-\frac{1}{x^2}}\left(\dfrac{1}{x^5} - \dfrac{3}{x^3}\right)$

2. (10 Points) Find the point on the curve $y = x^{3/2}$ that is closest to the point $(\frac{5}{2}, 0)$.

3. (10 Points) Find the length of the parametric curve

$$x = 3t^2, \quad y = 2t^3, \quad 0 \le t \le 1.$$

4. (10 Points) Determine whether the improper integral

$$\int_1^\infty \frac{1-\cos x}{x^2} dx$$

   is convergent or divergent.

5. (10 Points) Find the radius of convergence of the power series $\sum\limits_{n=1}^{\infty} n^2 x^n$ and evaluate $\sum\limits_{n=1}^{\infty} \dfrac{n^2}{2^n}$.

6. (10 Points) Find the equation of the tangent plane and the normal line to the surface

$$S : x^2 y + e^{xyz} - 2\cos(xz) = 0$$

   at $(1, 1, 0)$.

7. (10 Points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function. Assume that all second partial derivatives of f exist and continuous and

$$f_{xx} + f_{yy} = 0, \qquad \text{for all } (x, y) \in \mathbb{R}^2.$$

   Define $g : \mathbb{R}^2 \to \mathbb{R}$ by $g(u, v) = f(u^2 - v^2, 2uv)$. Find $g_{uu} + g_{vv}$.

8. (10 Points) Evaluate the double integral $\iint_D \sin(x+y)\,dA$ where D is the region bounded by $x + y = \pi$, $x + y = 0$ and $x - y = \pi$ and $x - y = 0$.

9. (10 Points) Evaluate the triple integral $\iiint_E \sqrt{x^2+y^2}\,dV$, where E is the solid region in $\mathbb{R}^3$ bounded by the surface $z=1$ and $z=x^2+y^2$.

10. (10 Points) Evaluate the line integral $\int_C (ye^x+\sin y)dx+(e^x+x\cos y)dy$ along the curve $C:\mathrm{r}(t)=(t^2+1)\mathrm{i}+(t^2-1)\mathrm{j}$, $0\le t\le 2$. Hint: you may find a potential function f of the vector field $\mathrm{F}(x,y)=(ye^x+\sin y)\mathrm{i}+(e^x+x\cos y)\mathrm{j}$, i.e. find f so that $\mathrm{F}=\nabla f$.