臺灣綜合大學系統 107 學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	A07、C11
		科目碼	A0702

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 1 頁

1. Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 2 & -1 & -1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ and $T(\mathbf{x}) = A\mathbf{x}$ for $\mathbf{x} \in \mathbf{R}^4$. Find bases for $\ker(T)$ and $\text{im}(T)$. (15%)

2. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

   (a) Find an invertible matrix P such that $P^{-1}AP$ is diagonal. (15%)

   (b) Find $A^5 - A^4 + 2A^3 - 3A^2 + 2A + I$. (10%)

3. If A is an $n \times n$ matrix such that $A^2 = A$. Let $\mathbf{U} = \{\mathbf{v} \in \mathbf{R}^n | A\mathbf{v} = \mathbf{v}\}$ and $\mathbf{W} = \ker(A)$. Show that $\mathbf{R}^n = \mathbf{U} \oplus \mathbf{W}$. (15%)

4. Let $B_2 = \{(1,0),(1,1)\}$ and $B_3 = \{(1,1,1),(1,0,1),(0,0,1)\}$ be ordered bases of $\mathbf{R}^2$ and $\mathbf{R}^3$, respectively. If $T(x,y) = (x-y, x+2y, 2x+y)$, find the matrix representation of T with respect to B_2 and B_3. (15%)

5. Let $S = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n\}$ be a subset of an n-dimensional vector space $\mathbf{V}$. Does S spans $\mathbf{V}$ imply that S is linearly independent? Why? (15%)

6. Let $\mathbf{V}$ be the space of all polynomials of degree at most 3. Define the inner product on $\mathbf{V}$ by $< p, q >= \int_0^1 p(x)\, q(x)\, dx$ for $p, q \in \mathbf{V}$. Let $\mathbf{S}$ be the subspace of $\mathbf{V}$ spanned by $\{1, x\}$. If $\mathbf{v} = x^2$, find a vector $\mathbf{u} \in \mathbf{S}$ and a vector $\mathbf{w} \in \mathbf{S}^{\perp}$ such that $\mathbf{v} = \mathbf{u} + \mathbf{w}$. (15%)