科目名稱	工程數學	類組代碼	**D09**
		科目碼	**D0992**

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 **1** 頁

1. Solve the initial-value problem $y'' + y = 4x + 10\sin x$, $y(\pi) = 0$, $y'(\pi) = 2$ by the method of undetermined coefficients. **(25%)**

2. (a) Find the Laplace transform of $\cos 5t$. **(5%)**

   (b) Find the inverse Laplace transform of $\dfrac{1}{s-1}$. **(5%)**

   (c) Use the Laplace transform to solve the initial-value problem
   $y' - y = 2\cos 5t$, $y(0) = 0$. **(15%)**

3. Given a matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

   (a) Find the eigenvalues of $\mathbf{A}$. **(5%)**

   (b) Find the orthonormal eigenvectors of $\mathbf{A}$. **(10%)**

   (c) Find $\sin(3\mathbf{A})$. **(10%)**

4. Verify Green's theorem by evaluating both integrals in
   $\oint_C 3x^2 y\, dx + (x^2 - 5y)\, dy = \iint_R (2x - 3x^2)\, dA$, where C is the rectangle with vertices (-1, 0), (1, 0), (1, 1), (-1, 1) and R is its interior region. **(25%)**