臺灣綜合大學系統107學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	D25
		科目碼	D2592

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 2 頁

1. (5 pts) Let A be the augmented matrix of the linear system

$$\begin{cases} x_1 + 2x_2 \quad\quad + x_4 = 1 \\ 2x_1 + 3x_2 - x_3 - x_4 = 3 \\ \quad\quad x_2 + x_3 - 3x_4 = -1 \end{cases}.$$

Assume that the reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & \alpha & 0 & 3 \\ 0 & 1 & 1 & 0 & \beta \\ 0 & 0 & 0 & 1 & \gamma \end{bmatrix}.$$

Find $\alpha + \beta - \gamma$.

2. (5 pts) Assume that $\begin{vmatrix} a & b & c \\ p & q & r \\ t & u & v \end{vmatrix} = -3$. Find $\begin{vmatrix} -p-t & -q-u & -r-v \\ 2a & 2b & 2c \\ p+a & q+b & r+c \end{vmatrix}$.

3. Let $A \in \mathbb{R}^{n\times n}$ and $\beta \in \mathbb{R}^{n\times 1}$. Let $A_p \in \mathbb{R}^{n\times n}$ $(1 \le p \le n)$ be defined as

$$(A_p)_{*i} = \begin{cases} A_{*i} & , \text{ if } i \neq p \\ \beta & , \text{ if } i = p \end{cases},$$

where $(A_p)_{*i}$ and A_{*i} denote the i-th column of A_p and A, respectively. Assume that A is invertible.

(a) (10 pts) Show that the linear system $Ax = \beta$ has a unique solution and the solution is $x = \begin{bmatrix} \frac{|A_1|}{|A|} \\ \frac{|A_2|}{|A|} \\ \vdots \\ \frac{|A_n|}{|A|} \end{bmatrix}$.

(b) (5 pts) Use (a) to solve the linear system $\begin{cases} 2x + 3y - 3z = 1 \\ -x + 2y + 2z = 0 \\ x \quad\quad + 3z = 5 \end{cases}$.

4. Let $W = \{A \in \mathbb{R}^{2\times 2} \mid A = -A^T\}$.

(a) (5 pts) Show that W is a subspace of $\mathbb{R}^{2\times 2}$.

(b) (5 pts) Find a basis for W.

5. Let $A \in \mathbb{R}^{m\times n}$, $N(A) = \{x \in \mathbb{R}^{n\times 1} \mid Ax = 0\}$ be the null space of A and $R(A) = \{Ax \mid x \in \mathbb{R}^{n\times 1}\}$ be the range of A.

(a) (5 pts) Let $N(A) = \mathbb{R}^{n\times 1}$. Show that A is the zero matrix.

(b) (10 pts) Let $\dim N(A) < n$. Show that $\dim N(A) + \dim R(A) = n$.

6. Let $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

(a) (10 pts) Find all eigenvalues of A and their corresponding eigenspaces.

(b) (5 pts) Find A^{10}.

　　背面有題，請繼續作答。

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7. Let $\vec{u}_1 = (2, 1, -1)$, $\vec{u}_2 = (1, 0, 1)$, $\vec{u}_3 = (0, 1, 1) \in \mathbb{R}^3$.

   (a) (5 pts) Show that $\beta = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is a basis of $\mathbb{R}^3$.

   (b) (10 pts) Apply the Gram-Schmidt process to β to find an orthonormal basis for $\mathbb{R}^3$.

8. Let $T : P_1(\mathbb{R}) \longmapsto \mathbb{R}^2$ be define by $T(p(x)) = (p(0), p(1))$.

   (a) (5 pts) Show that T is a linear transformation.

   (b) (5 pts) Show that T is $1-1$.

   (c) (5 pts) Find $T^{-1}(-2, 1)$.

   (d) (5 pts) Consider the bases $\beta = \{1, x\} \subseteq P_1(\mathbb{R})$, $\gamma = \{(1,1), (1,0)\} \subseteq \mathbb{R}^2$ for $P_1(\mathbb{R})$ and $\mathbb{R}^2$, respectively. Find $[T]_{\gamma,\beta}$(the matrix of T relative to the bases β and γ).