

臺灣綜合大學系統 108 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	共同考科
		科目碼	E0011
※本項考試依簡章規定各考科均「不可以」使用計算機		本科試題共計 2 頁	

Answer without complete work shown receives no credit. 所有計算過程都必須詳細列出，否則不予計分。

1. (10 points) Find the following limits.

(a)

$$\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}.$$

(b)

$$\lim_{x \rightarrow 0} \frac{x^2}{\sec x - 1}.$$

2. (10 points) Evaluate  $\frac{\partial f}{\partial x}\bigg|_{(0,0)}$  and  $\frac{\partial f}{\partial y}\bigg|_{(0,0)}$  for

$$f(x, y) = \begin{cases} \frac{3x^4 + xy^2}{2x^3 + 4xy + y}; & (x, y) \neq (0, 0) \\ 0; & (x, y) = (0, 0). \end{cases}$$

3. (10 points) Given

$$f(t) = \begin{cases} 1; & t \leq 0 \\ 1 - t; & t > 0 \end{cases}$$

and

$$F(x) = \int_{-1}^{2ax+2} f(t) dt$$

with  $a > 0$ , find  $a$  so that  $F$  is maximum at  $x = -2a$ .

4. (10 points) Find the largest possible area of a triangle with vertices  $(0, 2)$ ,  $(1, 0)$  and the third vertex on the ellipse

$$x^2 + \frac{y^2}{4} = 1.$$

5. (10 points) Evaluate

$$\int_{\ln \frac{1}{4}}^{\ln \frac{1}{2}} \frac{e^x}{\sqrt{1 - 4e^{2x}}} dx.$$

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6. (10 points) Evaluate

$$\int_0^{\frac{\sqrt{2}}{2}} \int_y^{\sqrt{1-y^2}} e^{x^2+y^2} dx dy.$$

7. (10 points) Derive the complete Taylor series expansion for

$$\ln\left(\frac{1+2x}{1-2x}\right)$$

about  $x = 0$ . (In the form  $\sum_{k=1}^{\infty} a_k x^{2k-1}$  with a general formula for  $a_k$ .)

8. (10 points) Given function  $T(x, y) = 1+x^2-y^2$ , find the curve  $\gamma(t) = (x(t), y(t))$  so that  $\gamma(0) = (1, 4)$  and  $\gamma'(t) = -\nabla T(\gamma(t))$ .

9. (10 points) Find  $(a, b)$  with  $-\frac{1}{2} \leq b \leq \frac{1}{2}$  so that

- The point  $P = (1, a, b)$  is on the surface  $E$  defined by

$$\frac{x}{2} - \frac{y}{4} + \frac{\sin(2z)}{4} = 0$$

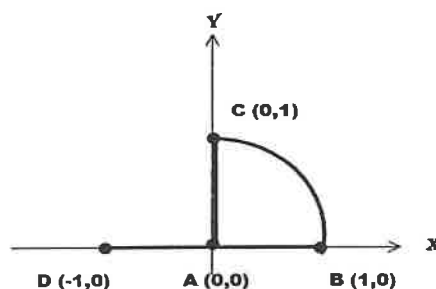
- The tangent plane to  $E$  at  $P$  contains lines

$$l_1(t) = P + t\left(\frac{1}{2}, 1, 0\right) \quad \text{and} \quad l_2(t) = P + t(0, 2, 1)$$

10. (10 points) Evaluate  $\int_L \vec{F} \cdot d\vec{r}$ , where

$$\vec{F} = (4x + 5y, e^{\cos y} + 7x)$$

and  $L$  is the path from  $A$  to  $B$ , to  $C$  to  $A$  and to  $D$  as shown below:



Note: The path from  $B$  to  $C$  is circular.