臺灣綜合大學系統108學年度學士班轉學生聯合招生考試試題

科目名稱	微積分C	類組代碼	共同考科
		科目碼	E0013

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計2頁

*** Answer without complete work shown receives no credit. No electronic devices allowed. ***

1. **(10 points)** Find the following limits:
   (a) (5 points) $\lim_{x\to 0} \dfrac{2^x-1}{4^x-1}$.
   (b) (5 points) $\lim_{x\to 0} \sin^{-1}(\dfrac{1-x}{1-x^2})$, where $\sin^{-1}$ is the inverse function of sine.

2. **(10 points)** Define $f(x)=\tan^2(x)$ for $x\in(0,\frac{\pi}{2})$ and let f^{-1} be its inverse function. Find $(f^{-1})'(3)$.

3. **(10 points)** Compute the following integrals
   (a) (5 points) $\int_0^{\pi/2} \dfrac{\sin 2x}{2+\cos x}dx$.
   (b) (5 points) $\int_1^2 \dfrac{(\ln x)^2}{x^3}dx$.

4. **(10 points)** Find the arc length of the curve with equation $x^{2/3}+y^{2/3}=1$ within the region $\{(x,y)\mid x\geq 0 \text{ and } y\geq 0\}$.

5. **(10 points)** Find the slope of the tangent line to the polar curve $r=1+\sin(2\theta)$ at the point specified by $\theta=\pi/3$.

6. **(10 points)** Find the radius of convergence of the series $\sum_{n=1}^{\infty} \dfrac{n(x+3)^n}{4^{n+1}}$.

7. **(10 points)** Find the Maclaurin series of order 4 for the function $f(x)=e^{-x^2}\cos x$, i.e., approximate $f(x)$ by a polynomial $a_0+a_1x+a_2x^2+a_3x^3+a_4x^4$.

科目名稱	微積分C	類組代碼	共同考科
		科目碼	E0013

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計2頁

8. (10 points) Find the maximum value of the function $f(x,y,z) = x+2y+3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.

9. (10 points) Let $\mathbf{F}(x,y) = \dfrac{-2y}{x^2+y^2}\mathbf{i} + \dfrac{2x}{x^2+y^2}\mathbf{j}$ and let $D = \{(x,y) \mid x^2 + y^2 = 9\}$. Find

$$\int_{\partial D} \mathbf{F} \cdot \mathbf{T} ds,$$

where we traverse the boundary ∂D in the counterclockwise direction and $\mathbf{T}$ is the unit tangent vector.

10. (10 points) Let $\mathbf{F}(x,y,z) = \dfrac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2+y^2+z^2)^{3/2}}$ and let $D = \{(x,y,z) \mid \dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{9} = 1\}$. Find the flux of $\mathbf{F}$ outward across the boundary of D.