臺灣綜合大學系統 108 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 2 頁

Answer without complete work shown receives no credits.

(1) (10 pts) Evaluate the followings.

(a) $\lim_{x\to 0} \frac{1}{\cos x}$.

(b) $\lim_{x\to 0} \frac{1-\cos x}{e^x - 1 - x}$.

(2) (10 pts) Water is pumped into a spherical balloon so that its volume increases at a rate $5cm^3/s$. How fast is the radius of the ballon increasing when the diameter is $4cm$? Here the volume of a ball of radius r is $\frac{4}{3}\pi r^3$.

(3) (10 pts) Let $f : (-\delta, \delta) \to \mathbb{R}$ be a function so that $f(0) = \frac{1}{2}$ for $\delta > 0$. Assume that $f(x)$ satisfies the equation

$$x^2 + (f(x))^2 = \left(2x^2 + 2(f(x))^2 - x\right)^2$$

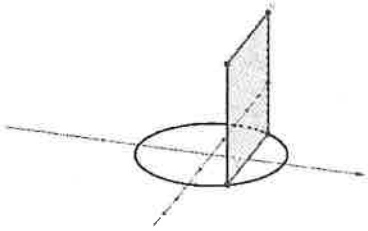
for all $x \in (-\delta, \delta)$. Suppose we know that f is differentiable at 0. Compute the tangent line to the curve $y = f(x)$ at the point $\left(0, \frac{1}{2}\right)$.

(4) (10 pts) Find the minimum of the function

$$f(x) = \int_1^{\sqrt{x}} \frac{e^t}{t} dt$$

on $[1, \infty)$. Explain how you obtain the minimum and find the point where the minimum of f occurs.

(5) (10 pts) Find the volume of the solid S where the base of S is a circular disk of radius 1 and parallel cross sections perpendicular to its base are squares.



(6) (10 pts) Evaluate the improper integral

$$\int_0^\infty \frac{x\tan^{-1} x}{(1+x^2)^2}dx$$

if it exists. Here $\tan^{-1} x = \arctan x$.

(7) (10 pts) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}x^n.$$

(8) (10 pts) Let f be the function

$$f(x,y) = \begin{cases} \dfrac{xy^2+\frac{1}{2}y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Evaluate $f_x(0,0)$ and $f_y(0,0)$ if they exist.

(9) (10 pts) Use Lagrange multipliers to find the extremum of the function

$$f(x,y,z) = xyz$$

subject to the constraint $xy + 2xz + 2yz = 12$.

(10) (10 pts) Let $a > 0$. Evaluate the double integral

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \cos(x^2+y^2)dydx.$$