

臺灣綜合大學系統 108 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D04
		科目碼	D0491

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 1 頁

1. The given function  $f(x)$  is assumed to have the period  $2\pi$ .

(a) Sketch  $f(x)$  for  $-\pi < x < \pi$ . (5 points)

(b) Find the Fourier series of  $f(x)$ . Show the details of your work. (10 points)

$$f(x) = \begin{cases} x + \pi, & -\pi < x < 0 \\ -x + \pi, & 0 < x < \pi \end{cases}$$

2. Find a general solution of the ODE (Ordinary Differential Equation). Show the details of your work. (10 points)

$$y'' + \omega^2 y = r(t), r(t) = \sin(t), \omega = 0.5, 1.2, 2, 10$$

3.  $y' = \frac{2x}{y}, y(1) = 5$ . (5 points)

4. Calculate  $\nabla^2 f$ , where  $f = 2e^{xyz}$ . (10 points)

5. Prove the following equations. (10 points)

$$(\vec{u} \times \vec{v})' = \vec{u}' \times \vec{v} + \vec{u} \times \vec{v}'$$

6. Diagonalize the matrix. (10 points)

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Using the Laplace transform and showing the details of your work, solve the IVP (Initial Value Problem). (15 points)

$$\begin{aligned} y_1' &= y_2 + 2 - u(t-1) \\ y_2' &= -y_1 + 1 - u(t-1) \\ y_1(0) &= 2 \\ y_2(0) &= 1 \end{aligned}$$

8. Is the following set of vectors linearly independent? Show the details of your work. (10 points)

$$[1 \ 0 \ 3], [1 \ 1 \ 1], [0 \ 1 \ 0]$$

9. For what values of  $a_1$  are  $[2 \ 0 \ a_1]$  and  $[1 \ 0 \ 8]$  orthogonal? (5 points)

10. Solve by Cramer's rule. Show details. (10 points)

$$\begin{aligned} 3x + y + 2z &= 1 \\ x - y + 3z &= -3 \\ y - 2z &= 1 \end{aligned}$$