臺灣綜合大學系統109學年度學士班轉學生聯合招生考試試題

科目名稱	微積分A	類組代碼	共同考科
		科目碼	E0011

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 2 頁

答題時，請詳述計算過程，否則將不予計分。

1. (10 pts) Evaluate the following limits if they exist.

   (a) $\lim_{n\to\infty}\frac{3n+2}{2n+1}$

   (b) $\lim_{x\to 0}\frac{\cos(2x)-1}{x^2}$

2. (10 pts) A curve in $\mathbb{R}^2$ is given parametrically by

$$x = t^2 + 2t + 3$$
$$y = t^4 - 3t^3$$

   for all $t > 0$. Find $\frac{dy}{dx}$ at the point $(6,-2)$.

3. (10 pts) Let $x > 0$ and ΔABC be a triangle whose side lengths are $\overline{BC} = 5$, $\overline{AC} = 4$, $\overline{AB} = 3$.

   Choose a point P on $\overline{AB}$, and a point Q on $\overline{BC}$, and a point R on $\overline{AC}$ so that

$$\frac{\overline{BP}}{\overline{AP}} = \frac{\overline{CQ}}{\overline{BQ}} = \frac{\overline{AR}}{\overline{CR}} = x$$

   Let $f(x)$ be the area of ΔPQR. Find the critical point and the minimum of $f(x)$.

4. (10 pts) Find the radius of the convergence of the power series $\sum_{n=0}^{\infty}\frac{(2n)^n}{n!}x^n$.

5. (10 pts) Evaluate the improper integral

$$\int_0^{\frac{\pi}{2}}\cos x\left(\ln\cos\frac{x}{2}+\ln\sin\frac{x}{2}\right)dx$$

6. (10 pts) Let $g:(0,\infty)\to\mathbb{R}$ be a twice differentiable function. Assume that

$$g(1)=1,\ g'(1)=3,\ g''(1)=-4.$$

   Define a real valued function h on $\mathbb{R}^3\setminus\{(0,0,0)\}$ by

$$h(x,y,z)=g\left(\sqrt{x^2+y^2+z^2}\right)$$

   Calculate $\frac{\partial^2 h}{\partial x^2}(P)+\frac{\partial^2 h}{\partial z^2}(P)+\frac{\partial^2 h}{\partial z^2}(P)$ where $P=\left(\frac{2}{3},\frac{2}{3},\frac{-1}{3}\right)$.

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7. (10 pts) Let S be the surface defined by the equation

$$x\cos(xy)+z^2y^4-7xz=1$$

and $P(0,1,1)$ be a point on S. Find an equation that defines the tangent plane to S at P and a parametric equation of the normal line to S at P.

8. (10 pts) Evaluate the double integral

$$\iint_R (y-x)\,dA$$

where $R=\{(x,y)\in\mathbb{R}^2 : 1\le x^2+y^2\le 4,\ x\ge 0\}$.

9. (10 pts) Let C be the curve in $\mathbb{R}^3$ defined by the parametric equation

$$x(t)=\cos(t),\ y(t)=\sin(t),\ z(t)=t$$

for $0\le t\le a$. Suppose that the arc length of C is $\dfrac{\sqrt{2}\pi}{4}$. Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ of the vector field $\mathbf{F}=xz\mathbf{i}+yz\mathbf{j}+x^3\mathbf{k}$ on $\mathbb{R}^3$.

10. (10 pts) Find the flux of the vector field $\mathbf{F}$ on $\mathbb{R}^3$ defined by

$$\mathbf{F}=3x\mathbf{i}+2y\mathbf{j}+5z\mathbf{k}$$

through the surface $S=\left\{\left(x,y,\sqrt{1-x^2-y^2}\right)\in\mathbb{R}^3 : x^2+y^2\le 1\right\}$ oriented with upward pointing normal vector field.

備註：$\mathbf{i}=(1,0,0)$, $\mathbf{j}=(0,1,0)$, $\mathbf{k}=(0,0,1)$.