科目名稱	基礎數學	類組代碼	D25
		科目碼	D2591

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 2 頁

I. Fill-in Problems (50 pts)

1. (10 pts)

(a)(2 pts) What is the name of the formula

$$\oint_C (Pdx + Qdy) = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\, dxdy,$$

where P and Q have continuous partial derivatives on an open region that contains $D \subset \mathbb{R}^2$, D is a region bounded by C, C is a positive oriented, piecewise-smooth, simple closed curve.

(b)(2 pts) What is the name of the formula

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S},$$

where $\mathbf{F}$ is a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains S, S is an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation.

(c)(2 pts) $\mathbf{F}(x, y, z) =< 2x, 3y, z^2 >$. What is div $\mathbf{F}$?

div $\mathbf{F}$ =____.

(d)(2 pts) $f(x)$ is continuously differentiable on $\mathbb{R}$. What is the formula to the arc length of the graph $y = f(x)$ from $x = a$ to $x = b$?

Arc length formula : ____.

(e)(2 pts) $f(x, y)$ is continuously differentiable on $\mathbb{R}^2$. What is the formula to the surface area of the surface $z = f(x, y)$ over domain D?

Surface area formula : ____.

2. (10 pts) Find a power series centered at 0, that is $\sum_{n=0}^{\infty} a_n x^n$, for the function $\frac{24x-35}{6x^2+29x-5}$.

Hint: $\frac{1}{1-x} = 1 + x + x^2 + ... + x^n + ...$ if $|x| < 1$.

Power series : ____.

3. (5 pts) Find the curvature $\kappa(t)$ of the curve $\gamma(t) =< 3t, 4\sin t, 4\cos t >$.

Hint: $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\gamma'(t)|} = \frac{|\gamma'(t) \times \gamma''(t)|}{|\gamma'(t)|^3}$, where $\mathbf{T}$ is the unit tangent vector.

$\kappa(t)$ =____.

4. (10 pts) Find the extreme values (if any) of $f(x, y, z) = x^2 + y^2 + z^2$ subject to both constraints $x + y + z = 1$ and $x + 2y + 3z = 6$.

   Extreme values=____.

5. (15 pts)

   (a)(5 pts) Let $f(x) = x^2$, $\forall x \in [0, 2] \equiv I$ and $S \equiv \{(x, y) | 0 \le y \le f(x) \text{ and } x \in I\}$. Find the volume of the solid generated by revolving S about $y-$axis.

   Volume=____.

   (b)(10 pts) Find the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$.

   Volume=____.

II. Writing Problems (50 pts)

6. (15 pts) Let

$$f(x) = \begin{cases} |x|^x & ,\text{if } x \neq 0 \\ 1 & ,\text{if } x = 0 \end{cases}.$$

   (a)(5 pts) Show that f is continuous at 0.

   (b)(10 pts) Show that f is not differentiable at 0.

7. (10 pts)

   (a)(5 pts) Prove the simple version of inverse function theorem. If f is a one-to-one differentiable function from $\mathbb{R}$ to $\mathbb{R}$ with continuous inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$.

   (b)(5pts) Show that any function of the form $z = f(x + at) + g(x - at)$ is a solution of the wave equation $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$, where $f, g \in C^2$ and a is a constant.

8. (10 pts) Derive the formula for triple integration in spherical coordinates $\iiint_R f(x, y, z) dV = \iiint_S f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi) \rho^2 \sin\phi \, d\rho d\theta d\phi$.

9. (15 pts) Let the sequence $\{a_n\}$ be $a_1 = 2$ and $a_n = \frac{1}{3 - a_{n-1}}$, $\forall n \ge 2$.

   (a)(10 pts) Use the Monotonic Sequence Theorem to show that $\{a_n\}$ is convergent.

   Hint: prove $\{a_n\}$ is bounded below and decreasing.

   (b)(5 pts) Find $\lim_{n \to \infty} a_n$.