臺灣綜合大學系統 111 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 C	類組代碼	共同考科
		科目碼	E0013

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 2 頁

1. (10 points)

   (a) (5 points) Evaluate the limit $\lim_{n\to\infty} \sqrt[n]{3^n + 5^n}$.

   (b) (5 points) Find the horizontal asymptote of the graph $y = x^2(2^{\frac{1}{x}} - 2^{\frac{1}{1+x}})$ for $x > 0$ if it exists.

2. (10 points) Define $f(x) = \dfrac{\int_{2x}^{x^2} \sqrt{1+\sin(\pi t)}\, dt}{x-2}$ for $x \neq 2$. Give a value of $f(2)$ such that f is continuous at 2.

3. (10 points) Let $f(x) = \frac{1}{3}x^3 + x + 1$ and $g = f^{-1}$ be the inverse function of f. A curve C satisfies the equation $2x^2y + xy^2 = 8$. Find a point (a, b) such that

   (i) (a, b) is in the first quadrant,

   (ii) (a, b) is on the graph of g, and

   (iii) the tangent line to the graph of g at (a, b) is perpendicular to the tangent line to the curve C at $(1, 2)$

4. (10 points) Let $f(x) = \ln(1 - x - 2x^2)$.

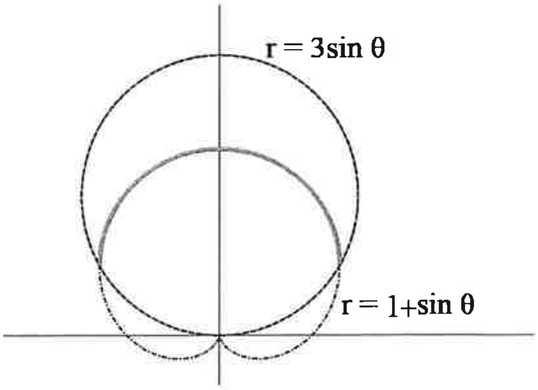
   (a) (5 points) Find the Taylor expansion for f about $x = 0$. (In the form $\sum_{k=0}^{\infty} a_k x^k$ with a general formula for a_k)

   (b) (5 points) Find the radius of convergence of the Taylor expansion in Problem(a).

5. (10 points) Let $u(x, y)$ be a differentiable function with $\frac{\partial u}{\partial x}(4, 1) = 1$ and $\frac{\partial u}{\partial y}(4, 1) = 2$. Suppose that $x = st$ and $y = \frac{s}{t}$ and define $h(s, t) = u\big(x(s, t), y(s, t)\big)$. At the point $(2, 2)$, find a unit vector $\mathbf{u}$ in the st-plane such that h increases most rapidly in the direction.

6. (10 points) Suppose that $f(\pi) = 4$ and $\int_0^{\pi} \left[f(x) + f''(x)\right] \sin x\, dx = 5$. Find $f(0)$.

7. (10 points) Find the arc length of the part of the curve $r = 1 + \sin\theta$ which is inside the curve $r = 3\sin\theta$ (the solid curve in the figure).



8. (10 points) Find the maximum of $f(x, y, z) = 2x + 7y - 3z$ on the ellipsoid $2x^2 + 7y^2 + 3z^2 = 6$.

9. (10 point) Evaluate the double integral

$$\iint_D e^{\frac{2x-y}{2x+y}}\, dA$$

where D is the trapezoid in the first quadrant with vertices $(2, 0)$, $(4, 0)$, $(0, 4)$ and $(0, 8)$.

10. (10 points) Let $\mathbf{F}(x, y, z) = \frac{x}{x^2+y^2+z^2}\mathbf{i} + \frac{y}{x^2+y^2+z^2}\mathbf{j} + \frac{z}{x^2+y^2+z^2}\mathbf{k}$. Compute the surface integral

$$\iint_S \mathbf{F} \cdot d\overrightarrow{S}$$

(using the outward pointing normal), when S is the surface $x^2 + y^2 + z^2 = 225$.