臺灣綜合大學系統113學年度學士班轉學生聯合招生考試試題

科目名稱	微積分B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 1 頁

There are 10 questions worth 10 points each.

Show all your works. Simplify and highlight your final answers.

Answers without work shown will NOT receive credits.

1. Given function $f(x) = (x^2 - x + 1)^{100}$. (a) Find the first derivative $f'(0)$.

   (b) Find the second derivative $f''(0)$.

2. Evaluate the limit. $\lim_{x \to 0} \dfrac{\sqrt{3 + \cos 3x} - 2}{x^2}$

3. (a) Given function $f(x) = 2^{x^2}$. Find the derivative $f'(2)$.

   (b) Given function $g(x) = x^{2^x}$. Find the derivative $g'(2)$.

4. Evaluate the definite integral. $\displaystyle\int_2^3 \frac{x}{x^2 - 5x + 4} dx$

5. Evaluate the improper integral. $\displaystyle\int_3^\infty e^{-\sqrt{3x}} dx$

6. Given the Taylor series of the function as below. Find the values of c_1, c_2, c_3.

$$\sqrt[5]{32 + x} = 2 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

7. (a) Given function $f(x) = \sin(3x)$. Find the higher derivative $f^{(111)}(0)$.

   (b) Given function $g(x) = \sin(x^3)$. Find the higher derivative $g^{(111)}(0)$.

8. Given function $f(x, y) = (4x^2 + y^2)e^{-2x}$. Find all critical points and determine their types (local maximum/local minimum/saddle point).

9. The Cobb-Douglas production function is $P(x, y) = x^{1/3} y^{2/3}$ (x: capital, y: labour) subject to budget constraint $3x^{1/2} + 5y^{1/2} = 45$. Use the method of Lagrange multiplier to find the values of x, y such that P is maximized.

10. Evaluate the double integral. $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \sqrt{x} \cdot \frac{\sin(y^2)}{y^2} \, dy \, dx$