科目名稱	微積分 A	類組代碼	共同考科
		科目碼	E0011

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計　1　頁

There are 10 questions worth 10 points each.

Show all your works. Simplify and highlight your final answers.

Answers without work shown will NOT receive credits.

1. Given functions $f(x) = \log_2(x^2)$ and $g(x) = \log_x(2^x)$. Find $f'(2)$ and $g'(2)$.

2. Evaluate the limit. $\lim\limits_{x \to 0} \dfrac{\sin(2\arcsin(3x)) - 6x}{x^3}$

3. Let function $g(x)$ be the inverse of $f(x) = x\sqrt{1 + 2x^2}$. Find $g'(6)$.

4. Evaluate the limit. $\lim\limits_{n \to \infty} \left[\dfrac{2\arctan(2n)}{\pi} \right]^n$

5. Evaluate the definite integral. $\displaystyle\int_0^{\pi/3} (3 + \tan\theta\sec\theta)^2 \, d\theta$

6. Given function $f(x) = (1-x)^3 e^{-x^4}$. Find the higher derivative $f^{(2025)}(0)$.

7. Given polar curve $r = e^{2\theta}$. Find the slope of tangent line of curve at $\theta = \dfrac{\pi}{4}$.

   Also find the arc length of curve for $0 \leq \theta \leq \dfrac{\pi}{2}$.

8. Let C be the curve of intersection of surfaces $xy + yz + zx = -14$ and $x^2 + y^2 + z^2 = 29$. The tangent line of curve C at point $(2, 3, -4)$ is given by $\dfrac{x-2}{a} = \dfrac{y-3}{b} = z + 4$. Find the values of a, b.

9. Use the method of Lagrange multiplier to find the shortest and longest distance from the origin to curve $9x^2 + 16xy + 21y^2 = 125$.

10. Let D be the region in xy-plane bounded by $x^2 = y$, $x^2 = 3y$, $y^2 = x$, $y^2 = 3x$. Use the transformation $u = \dfrac{x^2}{y}$, $v = \dfrac{y^2}{x}$ to evaluate the double integral. $\displaystyle\iint_D \dfrac{y^2}{x^4 + 3y^2} \, dA$