科目名稱	微積分B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計　一　頁

> There are **10 questions**, each worth **10 credit points**.
> - **Show all your work** clearly to receive full credit.
> - **Simplify** and **highlight** your final answers for easy review.
> - **Answers submitted without supporting work will NOT earn any credit.**

1. Let $g(x)$ be the function
$$g(x) = e^{x^{-2}}.$$
Compute $g'(1)$ and $g''(1)$.

2. Find the extreme values of the function
$$f(x) = \frac{x^2 - 3}{x^3}.$$
on the interval $[1, 5]$.

3. Evaluate the following definite integral:
$$\int_0^1 x e^{2x} dx.$$

4. Evaluate the following definite integral:
$$\int_0^1 \frac{1}{\sqrt{x^2 + 3}} dx.$$

5. Let $f(x) = 2x^{-1}$, and $T = \sum_{n=0}^{\infty} a_n (x - 2)^n$ the Taylor series of $f(x)$. What is a_{100}?

6. Suppose that x and y satisfy the equation
$$y^3 - x^2 = 4.$$
Find dy/dx and d^2y/dx^2 when $(x, y) = (2, 2)$.

7. Find the length L of the space curve given by
$$x(t) = 3\cos\sqrt{t+1}, \quad y(t) = 3\sin\sqrt{t+1} \quad \text{and} \quad z(t) = 4\sqrt{t+1}$$
for $t = 0$ to 1.

8. Let
$$f(x, y) = x + \sin(x + 2y).$$
Find the unit vector in the direction in which f increases most rapidly at the point $(0, 0)$ and give the rate of change of f in that direction.

9. Solve the following optimization problem using Lagrange multipliers:
$$f(x, y) = x^2 \quad \text{subject to} \quad x + y = 1.$$

10. Evaluate the double integral
$$\iint_R (x + y) dA,$$
where R is the region bounded by $y = x$ and $y = x^2$.