科目名稱	微積分 C	類組代碼	共同考科
		科目碼	E0013

Answer without complete work shown receives no credit.

1. (10 points) Find the following limits.

   (a)
   $$\lim_{h \to 1} \frac{\int_0^{h^2-1} e^{x^2-2}\, dx}{h-1}.$$

   (b)
   $$\lim_{x \to 0^+} \frac{|x - \sin(3x)|}{7x}.$$

2. (10 points) Find point(s) (x_0, y_0) on the curve

   $$\sqrt{x} + \sqrt{y} = 12$$

   at which normal line(s) to this curve has(have) slope(s) $\frac{1}{3}$.

3. (10 points) Find the exact value of the infinite sum

   $$\sum_{k=3}^{\infty} \frac{(k+1)(-1)^k}{k!}.$$

4. (10 points) Two particles, A, B, initially start from origin $(0,0)$, move along two straight lines with angle $\frac{\pi}{6}$ between them. Particle A is moving at a speed of 2, and particle B is moving at a speed of 6. What is the rate of change of the distance between two particles, when particle A is 1 unit from the starting point?

5. (10 points) Given the function

   $$f(x, y, z) = 2xe^y - x \sin z,$$

   and point $P = (1, 0, 0)$, find a, b so that the rate of change of f at P along the unit vector

   $$\mathbf{u} = (a, 0, b)$$

   is 0. Here, $a > 0$.

6. (10 points) Find the maximum possible volume for a rectangular box, with edges parallel to coordinate axes, that is inscribed in the ellipsoid

$$\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1.$$

7. (10 points) Find all the saddle point(s) of

$$f(x,y) = e^y(y^2 - x^2).$$

8. (10 points) Evaluate the following integral

$$\int_0^1 \int_0^{\sqrt{x-x^2}} \sqrt{x^2 + y^2}\, dydx.$$

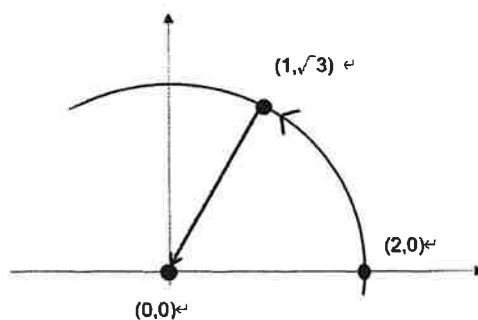(The formula $\cos^3\theta = \frac{\cos(3\theta)+3\cos\theta}{4}$ may be useful.)

9. (10 points) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where

$$\mathbf{F} = \left(\ln(y^2+1) - 4y + e^x, \frac{2xy}{y^2+1} + x \right)$$

and C is the path connecting $(2,0)$ to $(1,\sqrt{3})$ through the circle $x^2 + y^2 = 4$, and then to the origin $(0,0)$ by straight line. See the figure below:

背面有題，請繼續作答。

10. (10 points) Evaluate the upward flux

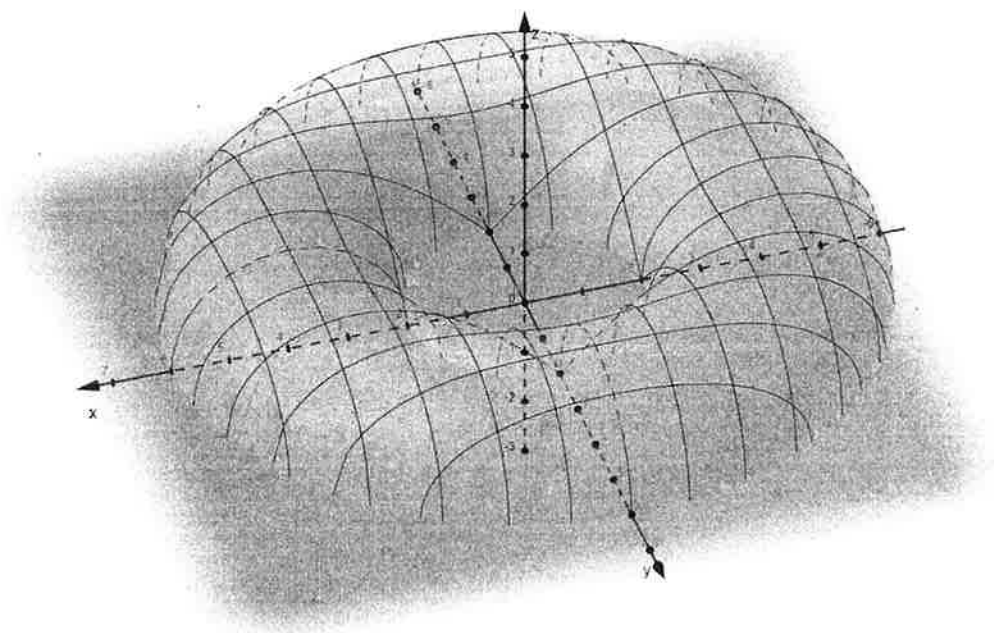$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where

$$\mathbf{F} = \left(\cos(yz^2) + 5x, \ln(x^2 + 2z^2 + 1) - 6y, z - \cos(x^2 + y^2)\right),$$

and S is the upper half (i.e. $z \geq 0$ part) of the torus

$$\left(\sqrt{x^2 + y^2} - 4\right)^2 + z^2 = 4$$

as shown below

(Figure generated using GeoGebra)