臺灣綜合大學系統 114 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D37
		科目碼	D3792

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 2 頁

1. (8%) If $\mathbf{A}^{-1}=\begin{bmatrix}-18 & 7\\ -5 & 2\end{bmatrix}$ what is $\mathbf{A}$?

   (A) $\mathbf{A}=\begin{bmatrix}\frac{-1}{18} & \frac{-1}{5}\\ \frac{1}{7} & \frac{1}{2}\end{bmatrix}$, (B) $\mathbf{A}=\begin{bmatrix}\frac{-1}{18} & \frac{-1}{7}\\ \frac{1}{5} & \frac{1}{2}\end{bmatrix}$, (C) $\mathbf{A}=\begin{bmatrix}\frac{18}{20} & \frac{-5}{20}\\ \frac{7}{20} & \frac{-2}{20}\end{bmatrix}$, (D), $\mathbf{A}=\begin{bmatrix}2 & -5\\ 7 & -18\end{bmatrix}$

   (E) $\mathbf{A}=\begin{bmatrix}-2 & 7\\ -5 & 18\end{bmatrix}$.

2. (8%) $\mathbf{A}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ find $\mathbf{A}^{-1}$?

   (A) $\mathbf{A}=\begin{bmatrix}\frac{1}{\cos\theta} & \frac{1}{\sin\theta} & 0\\ \frac{1}{\sin\theta} & \frac{1}{\cos\theta} & 0\\ 0 & 0 & 1\end{bmatrix}$, (B) $\mathbf{A}=\begin{bmatrix}-\sin\theta & \cos\theta & 0\\ \cos\theta & -\sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$, (C) $\mathbf{A}=\begin{bmatrix}\frac{1}{\sin\theta} & \frac{1}{\cos\theta} & 0\\ \frac{1}{\cos\theta} & \frac{1}{\sin\theta} & 0\\ 0 & 0 & 1\end{bmatrix}$,

   (D) $\mathbf{A}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$, (E) $\mathbf{A}=\begin{bmatrix}1 & 0 & \cos\theta\\ 0 & 1 & \sin\theta\\ \cos\theta & \sin\theta & 1\end{bmatrix}$.

3. (8%) Evaluate $\mathcal{L}\{f(t)\}, f(t)=\begin{cases}0 & 0<t<1\\ 3t-3 & t>1\end{cases}$

   (A) $\frac{2}{s^2}e^{-s}$, (B) $\frac{3}{s^2}e^{-s}$, (C) $\frac{3}{s^2}e^{s}$, (D) $\frac{2}{s^2}e^{s}$, (E) $\frac{6}{s^2}e^{s}$.

4. (6%) Find the div(curl $\mathbf{F}$)， $\mathbf{F}(x,y,z)=xyz\mathbf{i}+4x^2yz\mathbf{j}+2x^3z\mathbf{k}$.

   (A) no answer, (B) $\langle 5y-9x^2, 2xz-5z, -x^2\rangle$, (C) $\mathbf{0}$, (D) 0, (E) 1.

5. (8%) The given integral $\int_{(0,0)}^{(3,2)}(x+y+\frac{ky}{x^2+y^2})dx+(x^2-y+\frac{kx}{x^2+y^2})dy$ is independent or dependent of the path. Evaluate $k=0$.

   (A) No exist, (B) independent and $\frac{26}{3}-\frac{\pi}{4}$, (C) dependent and $\frac{26}{3}-\frac{\pi}{4}$, (D) independent and $\frac{26}{3}+\frac{\pi}{4}$,

   (E) dependent and $\frac{26}{3}+\frac{\pi}{4}$.

Find the LU-factorization of $\mathbf{A}=\begin{bmatrix}1 & 3 & 1\\ 1 & 1 & -1\\ 1 & 2 & 1\end{bmatrix}$ if possible.

6. (6%) $\mathbf{L}$:

(A) $\mathbf{L}=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 1 & 1 & 1\end{bmatrix}$, (B) $\mathbf{L}=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 2 & 1\end{bmatrix}$, (C) $\mathbf{L}=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 0.5 & 1\end{bmatrix}$, (D) $\mathbf{L}=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 1 & 4 & 1\end{bmatrix}$, (E) do not exist.

7. (6%) $\mathbf{U}$:

(A) $\mathbf{U}=\begin{bmatrix}1 & 3 & 1\\ 0 & -2 & -2\\ 0 & 0 & 1\end{bmatrix}$, (B) $\mathbf{U}=\begin{bmatrix}1 & 2 & -3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$, (C) $\mathbf{U}=\begin{bmatrix}1 & 0.5 & 3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$, (D) $\mathbf{U}=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 4\\ 0 & 0 & 1\end{bmatrix}$, (E) do not exist.

8. (6%) A particle whose position vector is $\mathbf{r}(t)=\cos t\mathbf{i}+\cos t\mathbf{j}+\sqrt{2}\sin t\mathbf{k}$ move with constant speed

(A) $\cos 2$, (B) 2, (C) $\sin 2$, (D) $\sqrt{2}$, (E) $\sin\sqrt{2}$.

9. (8%) If $\mathbf{F}=4y\mathbf{i}+6x\mathbf{j}$ and C is given by $x^2+y^2=1$, evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$

(A) 2π, (B) 4π, (C) 6π, (D) 10π, (E) 0.

10. (8%) A particle whose position vector is $\mathbf{r}(t)=\sin t\mathbf{i}+\cos t\mathbf{j}+3t\mathbf{k}$. Find the curvature

(A) $\cos 3t$, (B) 1, (C) 0.1, (D) 3, (E) $\sin 3t$.

11. (8%) Find the rank of the given matrix $\begin{cases}x_1-2x_2+3x_3=4\\ 2x_1-4x_2+6x_3=8\\ 3x_1+x_2+6x_3=5\end{cases}$ and determinate how many solutions this system has.

(A) 2 and infinity solution, (B) 2 and unique solution, (C) 3 and infinity solution, (D) 3 and unique solution, (E) no solution.

12. (6%) Diagonalize $\mathbf{A}=\begin{bmatrix}1 & 4\\ -1 & 1\end{bmatrix}$ if possible.

(A) $\mathbf{D}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, (B) $\mathbf{D}=\begin{bmatrix}-i & 0\\ 0 & -i\end{bmatrix}$, (C) $\mathbf{D}=\begin{bmatrix}1-2i & 0\\ 0 & 1+2i\end{bmatrix}$, (D) $\mathbf{D}=\begin{bmatrix}2i & -2i\\ 1 & 1\end{bmatrix}$, (E) do not exist.

Evaluate the double integral $\int_0^1\int_0^{1-x} e^{(y-x)/(y+x)}dxdy$,

13. (6%) Find the Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$

(A) $-\frac{1}{2}$, (B) $\begin{bmatrix}-2x & 2y\\ x & y\end{bmatrix}$, (C) $2(x^2+y^2)$, (D) $\dfrac{-1}{2(x^2+y^2)}$ (E) -2.

14. (8%) Evaluate the given integral

(A) $\frac{1}{6}(e^{-1}+e)$, (B) $\frac{1}{4}(e-e^{-1})$, (C) $\frac{1}{6}(e-e^{-1})$, (D) 0, (E) $\frac{1}{4}(e^{-1}+e)$.