

臺灣綜合大學系統 115 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	共同考科
		科目碼	E0011

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 1 頁

There are 10 questions, each worth 10 credit points.

- Show all your work clearly to receive full credit.
- Simplify and highlight your final answers for easy review.
- Answers submitted without supporting work will NOT earn any credit.

(1) Find $\lim_{x \rightarrow 4} \frac{4-x}{5-\sqrt{x^2+9}}$.

(2) If f' is continuous, $f(2) = 0$, $f'(2) = 7$, evaluate $\lim_{x \rightarrow 0} \frac{f(2+3x)+f(2+5x)}{x}$.

(3) Calculate $\int_{-1}^0 \frac{e^x}{e^x+e^{-1-x}} dx$.

(4) Determine the series convergent or divergent: $\sum_{k=1}^{\infty} \frac{k!}{k^k}$.

(5) Check the limit exist or not: $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2y+yz^2+x^2z^2}{x^4+y^2+z^4}$.

(6) Calculate the double integral $\int \int_D (3x+4y^2) dA$, where $D : 1 \leq x^2+y^2 \leq 4$, $y \geq 0$.

(7) Let $f(x, y) = \ln(x^2 + y^2)$. At the point $(1, 1)$, the directional derivative of the function f along a certain unit vector $\vec{u}$ is $\sqrt{2}$. Find the unit vector $\vec{u}$.

(8) Consider the polar curve defined by the equation $r^2 = 2 \cos(2\theta)$. Find the polar coordinates (r, θ) of the point(s) in the first quadrant where the tangent line to the curve is horizontal.

(9) Let $f(x) = e^x + \ln(x+1)$. This function is strictly increasing on the interval $[0, 1]$, and its inverse function is denoted as $f^{-1}(x)$. Calculate the integral: $\int_0^1 f(x) dx + \int_1^{e+\ln 2} f^{-1}(x) dx$.

(10) Let S be the sphere with center 0 and radius 2, calculate $\iint_S \text{curl } \mathbf{F} \cdot n dS$, where $\mathbf{F}(x, y, z) = (2x - 2z, x^2y, -xz^2)$ is a vector function.