

臺灣綜合大學系統 115 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。

本科試題共計 1 頁

There are 10 questions, each worth 10 credit points.

- Show all your work clearly to receive full credit.
- Simplify and highlight your final answers for easy review.
- Answers submitted without supporting work will NOT earn any credit.

1. Evaluate the following limites.

(a) (5 points) Evaluate $\lim_{x \rightarrow 1} \frac{\log_2(2x-1) - x^2}{x+3}$

(b) (5 points) Evaluate $\lim_{x \rightarrow 0} \frac{\log_3(2x+1) - x^2}{\sin x}$

2. Let $f(x) = \frac{x^2}{x-1}$ defined over $(-\infty, 1) \cup (1, \infty)$.

(a) (5 points) Find the intervals on which f is increasing.

(b) (5 points) Find the intervals on which f is concave down.

3. Compute the indefinite integral

$$\int 3x^3 + 3^x + \sin(3x) dx.$$

4. Evaluate $\int_1^3 \frac{\ln u}{u^2} du$.

5. For the plane curve defined by the equation

$$x^2 \cos(\pi y) + xy + 2 = 0,$$

find the tangent line equation at $(1, -1)$.

6. Find $f^{(72)}(0)$ of the function $f(x) = \cos(x^3)$.

7. Find all saddle points of the function $f(x, y) = xy - x^2y - xy^2$.

8. Use the method of Lagrangian multiplier to find the minimum of $f(x, y, z) = 2x + y - 3z$ subject to the constraint $z = x^2 + y^2$.

9. Find the unit tangent vector and the exact length of the curve

$$C: \mathbf{r}(t) = t^2\mathbf{i} + 6t\mathbf{j} + 4t^{3/2}\mathbf{k}, \quad 0 \leq t \leq 1.$$

10. Evaluate the double integral

$$\iint_D y^2 e^{xy} dA,$$

where D is the region enclosed by $y = x$, $y = 4$, and $x = 0$.