臺灣綜合大學系統115學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	A07.C11
		科目碼	A0702

※本項考試依簡章規定所有考科均「不可」使用計算機。　本科試題共計 1 頁

1. (5%) Let V be a vector space over a field F. State the definition of "V is a vector space over F".

2. (5%) Does the set $\{(x, y) \in R^2 \mid y = x + 1\}$ form a subspace of the vector space R^2? Justify your assertion.

3. (5%) Let $(V, \langle\cdot,\cdot\rangle)$ be an inner product space. State the definition of "$\langle\cdot,\cdot\rangle$ is an inner product".

4. (10%) Suppose that you know the following fact: "$\int_0^1 f(x)dx = 0 \Rightarrow f(x) = 0$ for all $x \in [0,1]$". Let $C[0,1]$ be the collection of real-valued continuous functions defined on $[0,1]$. Define $\langle\cdot,\cdot\rangle: C[0,1] \to R$ by $\langle f, g\rangle := \int_0^1 f(x)g(x)dx$. Show that $\langle\cdot,\cdot\rangle$ defines an inner product on $C[0,1]$.

5. (5%) Let V and W be vector spaces over a field F. Suppose that $T: V \to W$ is a linear transformation. Write down the definition of "T is a linear transformation".

6. (5%) Let $f: R \to R$ be defined by $f(x) = x^2$. Is f linear? Justify your assertion.

7. (10%) Let V and W be vector spaces over a field F. Let $T: V \to W$ be a linear transformation. Show that $N(T) := \{v \in V \mid T(v) = 0_W\}$ is a subspace.

8. (5%) Let V and W be vector spaces over a field F. Suppose that $T: V \to W$ is a linear transformation. Let $v \in V$ be an eigenvector of T. Write down the definition of "v is an eigenvector of T".

9. (20%) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.(i) (10%) Find the eigenvalues of A; (ii) (10%) Find a matrix Q such that $Q^{-1}AQ = D$, where D is the diagonal matrix consists of eigenvalues of A.

10. (10%) Let $P_2(R)$ denote the collection of real-coefficient polynomials of degrees at most 2. Define $T: P_2(R) \to P_2(R)$ by $T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2$. Find the matrix representation of T with respect to the standard ordered basis of $P_2(R)$.

11. (10%) Show that all the eigenvalues of a real symmetric matrix are real.

12. (10%) Let A and B be 4×4 matrices. Suppose that $\det(A) = 3$ and that $\det(B) = -2$. Find the following values with appropriate details. (i) (5%) $\det(AB)$; (ii) (5%) $\det(B^{-1})$.