科目名稱	微積分C	類組代碼	共同考科
		科目碼	E0013

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 2 頁

There are 10 questions, each worth 10 credit points.

- Show all your work clearly to receive full credit.
- Simplify and highlight your final answers for easy review.
- Answers submitted without supporting work will NOT receive any credit.

1. (10 pts) Define

$$f(x) = \frac{\displaystyle\int_{3x}^{x^2} \ln(1+t^2)\,dt}{x^2-3x}, \qquad x \neq 0,\ x \neq 3.$$

Evaluate $\lim_{x\to 0} f(x)$.

2. (10 pts)

(a) (5 pts) The curve is defined implicitly by $x^3+y^3=6xy$ (the *Folium of Descartes*). Find the equation of the tangent line to the curve at the point $(3,3)$.

(b) (5 pts) Water is pumped into an inverted right circular cone at a rate of $2\,\mathrm{m}^3/\mathrm{min}$. The cone has height 6 m and top-circle radius 3 m. Find the rate at which the water level is rising when the depth of water is 2 m.

3. (10 pts)

(a) (5 pts) Evaluate

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx.$$

(b) (5 pts) Given $f(0)=1$, $f'(0)=2$, and $\displaystyle\int_0^{\pi} [f(x)+f''(x)] \sin x\,dx = 3$, find $f(\pi)$.

4. (10 pts) Determine the interval of convergence (including endpoint analysis) of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-2)^n}{n\cdot 3^n}.$$

5. (10 pts) Let $f(x) = \dfrac{x}{(1+x^2)^2}$.

(a) (6 pts) Find the Maclaurin series for $f(x)$ (write the general term and state the radius of convergence).

(b) (4 pts) Using the result of part (a), find $f^{(7)}(0)$ (the value of the seventh derivative at $x=0$).

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6. (10 pts)

 (a) (7 pts) Using the method of Lagrange multipliers, find the extreme values of $f(x, y, z) = x^4 + y^4 + z^4$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

 (b) (3 pts) Find the extreme values of $f(x, y, z) = x^4+y^4+z^4$ described by the inequality $x^2 + y^2 + z^2 \leq 1$.

7. (10 pts) Let $f(x, y, z) = x^2z - y^2z + xy$ and $P = (1, -1, 2)$.

 (a) (4 pts) Compute the gradient $\nabla f(P)$.

 (b) (3 pts) Find the directional derivative of f at P in the direction of $\mathbf{v} = (1, 2, -2)$.

 (c) (3 pts) Find the maximum directional derivative of f at P, and the corresponding unit direction vector.

8. (10 pts) Evaluate the double integral

$$\int_{-2}^{0} \int_{0}^{\sqrt{4-x^2}} e^{x^2+y^2}\, dy\, dx.$$

9. (10 pts)

 (a) (5 pts) Use Green's Theorem to evaluate the line integral

$$\oint_C dx + \left(x + e^{\sin y}\right) dy,$$

 where C consists of the line segment from $(-1, 0)$ to $(1, 0)$ followed by the upper arc of the ellipse $\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$ from $(1, 0)$ back to $(-1, 0)$, traversed counterclockwise.

 (b) (5 pts) Evaluate the line integral

$$\int_\gamma dx + \left(x + e^{\sin y}\right) dy,$$

 where γ is the upper arc of the ellipse $\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$ from $(-1, 0)$ to $(1, 0)$.

10. (10 pts) Let

$$\mathbf{F}(x, y, z) = \left(x^3 + \sin(yz),\ y^3 + e^{xz},\ z^3 + \ln(x^2 + y^2 + 1)\right),$$

and let S be the sphere $x^2+y^2+z^2 = 9$ with outward-pointing normal. Use the Divergence Theorem to evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$