臺灣綜合大學系統 115 學年度學士班轉學生聯合招生考試試題

科目名稱	電磁學	類組代碼	D14
		科目碼	D1491

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 4 頁

Instructions: Unless otherwise stated, use SI units. You may leave your answers in terms of ϵ_0, μ_0, ϵ, μ, c, and π. For multiple-choice questions, choose the single best answer.

Part I. Multiple-Choice Questions 請於答案卡上作答，否則不予計分。

20 points total; 2 points each

1. In a charge-free region, the scalar potential V satisfies Laplace's equation,

$$\nabla^2 V = 0.$$

Which statement is most accurate?

A. V must be constant everywhere in the region.
B. V may have an isolated strict local maximum inside the region.
C. V may have an isolated strict local minimum inside the region.
D. Any maximum or minimum of V, if present, occurs on the boundary.
E. ∇V must vanish everywhere.

2. In electrostatics, the uniqueness theorem is most directly used to justify which statement?

A. Knowing V at one point determines V everywhere.
B. If V is specified on the boundary, then the solution to Laplace's or Poisson's equation is unique.
C. Knowing only the total charge determines the full potential distribution.
D. Any dimensionally correct function can be used as the electrostatic potential.
E. The surface charge density on a conductor must be uniform.

3. A point charge q is located at $z = d$ above a grounded infinite conducting plane at $z = 0$. For the region $z > 0$, the method of images replaces the conductor by

A. an image charge $+q$ at $z = -d$.
B. an image charge $-q$ at $z = -d$.
C. an image charge $-q$ at $z = 0$.
D. no image charge, because a grounded conductor does not affect the upper half-space.
E. an infinite sequence of image charges.

4. In the semi-infinite rectangular region

$$0 < x < a, y > 0,$$

suppose V satisfies

$$\nabla^2 V = 0, V(0,y) = V(a,y) = 0, V(x,y) \to 0 \text{ as } y \to \infty.$$

Which separated form is appropriate?

A. $\cos(n\pi x/a)e^{+n\pi y/a}$
B. $\sin(n\pi x/a)e^{-n\pi y/a}$
C. $e^{inx}e^{iny}$
D. $J_n(x)Y_n(y)$
E. $P_n(\cos\theta)/r^{n+1}$

5. At the interface between two linear dielectric media, assume there is no free surface charge. Which electrostatic boundary conditions are correct?

A. $E_{\perp}$ is continuous, and $D_{\parallel}$ is continuous.
B. $D_{\perp}$ is continuous, and $E_{\parallel}$ is continuous.
C. V must be discontinuous.
D. $\partial V / \partial n$ is always continuous.
E. $\mathbf{P} = 0$ in both media.

6. In a source-free region of vacuum, Maxwell's equations imply that the electric field satisfies

A. $\nabla^2 \mathbf{E} = 0$
B. $\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$
C. $\nabla^2 \mathbf{E} = -\rho / \epsilon_0$
D. $\nabla \times \mathbf{E} = 0$
E. $\partial \mathbf{E} / \partial t = 0$

7. A monochromatic plane electromagnetic wave in vacuum propagates in the $+z$direction. Its electric field is

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t)\, \hat{\mathbf{x}}.$$

What is the corresponding magnetic field?

A. $\mathbf{B}(z,t) = \frac{E_0}{c} \cos(kz - \omega t)\, \hat{\mathbf{y}}$
B. $\mathbf{B}(z,t) = cE_0 \cos(kz - \omega t)\, \hat{\mathbf{y}}$
C. $\mathbf{B}(z,t) = \frac{E_0}{c} \cos(kz - \omega t)\, \hat{\mathbf{z}}$
D. $\mathbf{B}(z,t) = \frac{E_0}{c} \cos(kz - \omega t)\, \hat{\mathbf{x}}$
E. $\mathbf{B} = 0$

8. For a plane electromagnetic wave in a vacuum, which statement is correct?

A. **E**, **B**, and the propagation direction are all parallel.
B. **E** and **B** are always 90°out of phase.
C. $\mathbf{E} \perp \mathbf{B}$, and the energy flux is in the direction of $\mathbf{E} \times \mathbf{B}$.
D. **B** determines the wave speed, but **E** does not.
E. Electromagnetic waves can propagate only inside conductors.

9. If one uses the unmodified Ampère's law,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

in time-dependent situations, the law conflicts most directly with

A. conservation of energy.
B. conservation of charge and the continuity equation.
C. Newton's third law.
D. Laplace's equation.
E. Coulomb's law.

10. For an electromagnetic wave incident on an ideal conductor, the electric field inside the conductor is zero. The boundary condition on the tangential electric field at the surface mainly implies that

A. the incident wave is completely transmitted.
B. a reflected wave is produced so that the total tangential electric field at the surface vanishes.
C. the magnetic field must vanish everywhere.
D. the normal component of the electric field must vanish.
E. the wave speed becomes infinite.

Part II. Questions and Calculation Problems 請於答案卷上作答，否則不予計分。

80 points total

11. [8 points] Consider a charge-free cavity enclosed by a conductor. Suppose the conductor is held at a constant potential, which may be taken as $V = 0$.

Answer the following:

(a) State the differential equation satisfied by V inside the cavity.

(b) Explain why the solution is unique once V is specified on the boundary.

(c) Show that $V = 0$everywhere inside the cavity.

(d) Explain why the electric field inside the empty cavity must vanish.

12. [12 points] In a source-free region of vacuum,

$$\rho = 0, \mathbf{J} = 0.$$

Answer the following:

(a) Write Maxwell's equations in differential form for this region.

(b) Starting from Faraday's law and the Maxwell-Ampère law, derive the wave equation for **E**.

(c) Write the corresponding wave equation for **B**.

(d) Explain physically how a time-varying electric field and a time-varying magnetic field can sustain propagating electromagnetic wave.

(e) Identify the wave speed in terms of ϵ_0 and μ_0.

13. [20 points] In the rectangular region

$$0 < x < a, 0 < y < b,$$

the scalar potential $V(x, y)$satisfies

$$\nabla^2 V = 0.$$

The boundary conditions are

$$V(0, y) = 0, V(a, y) = 0, V(x, 0) = 0,$$

and

$$V(x, b) = V_0 \sin\left(\frac{\pi x}{a}\right) + 2V_0 \sin\left(\frac{3\pi x}{a}\right).$$

Answer the following:

(a) Use separation of variables, $V(x, y) = X(x)Y(y)$, to derive the ordinary differential equations for $X(x)$and $Y(y)$.

(b) Find the eigenfunctions compatible with the boundary conditions at $x = 0$, $x = a$, and $y = 0$.

(c) Determine $V(x, y)$.

(d) Find the electric field

$$\mathbf{E}(x, y) = -\nabla V.$$

(e) Use the uniqueness theorem to explain why your solution is the physical solution.

14. [20 points] A linear dielectric sphere of radius $R$and permittivity $\epsilon$is placed in vacuum. Far from the sphere, the applied electric field is uniform:

$$\mathbf{E}_0 = E_0\hat{\mathbf{z}}.$$

There is no free surface charge at the boundary. Assume axial symmetry and use the following forms:

$$V_{\text{out}}(r, \theta) = -E_0 r\cos\theta + \frac{A\cos\theta}{r^2}, r > R,$$
$$V_{\text{in}}(r, \theta) = -Cr\cos\theta, r < R.$$

Answer the following:

(a) Explain why both V_{out} and V_{in} satisfy Laplace's equation rather than Poisson's equation.

(b) State the two electrostatic boundary conditions at $r = R$.

(c) Determine the constants A and C.

(d) Find the electric field inside the sphere, $\mathbf{E}_{\text{in}}$.

(e) Find the bound surface charge density $\sigma_b(\theta)$ on the dielectric sphere.

15. [20 points] In a source-free region of vacuum, a monochromatic plane wave has electric field

$$\mathbf{E}(z, t) = E_0\cos(kz - \omega t)\,\hat{\mathbf{x}}.$$

Answer the following:

(a) Use Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

to determine the corresponding magnetic field $\mathbf{B}(z, t)$, including its direction.

(b) Substitute the fields into the Maxwell-Ampère law,

$$\nabla \times \mathbf{B} = \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t},$$

and derive the dispersion relation between $\omega$and k.

(c) Find the instantaneous Poynting vector,

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}.$$

(d) Find the time-averaged intensity $\langle S \rangle$.

(e) Explain why $\mathbf{E}$, $\mathbf{B}$, and the propagation direction are mutually perpendicular in this wave.