

臺灣綜合大學系統 115 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D37
		科目碼	D3792

※本項考試依簡章規定所有考科均「不可」使用計算機。

本科試題共計 1 頁

1. Find an explicit solution, $y(x)$, to the differential equation

$$e^y \left(\frac{dy}{dx} + 1 \right) = e^x. \quad (20\%)$$

2. Consider $\mathbf{i}$ and $\mathbf{j}$ the standard basis for the system of two-dimensional vectors. Given the vector field

$$\mathbf{F}(x, y) = (e^x \sin(y) + e^{-y}) \mathbf{i} + (e^x \cos(y) - x e^{-y}) \mathbf{j},$$

- a) show that $\mathbf{F}(x, y)$ is conservative. (8%)
 b) Find a potential function ϕ that satisfies the relation $\mathbf{F} = \nabla \phi$. (8%)
 c) Evaluate

$$\int_{(0,0)}^{(1,\pi/2)} \mathbf{F} \cdot d\mathbf{r}. \quad (4\%)$$

3. The matrix

$$\mathbf{P} = \begin{pmatrix} a & 0 & \frac{4}{3\sqrt{2}} \\ b & \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} \\ \frac{2}{3} & c & -\frac{1}{3\sqrt{2}} \end{pmatrix}$$

is orthogonal and contains three missing values a , b , and c .

- a) Find a , b , and c . (10%)
 b) Show that $\mathbf{P}^T \mathbf{P} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. (10%)

4. Given a matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 2 \\ -7 & 8 \end{pmatrix},$$

use the properties of Eigenvalues and Eigenvectors to show that

$$\mathbf{A}^n \begin{pmatrix} 3 \\ 9 \end{pmatrix} = 6^n \begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

where n is a positive integer. (20%)

5. Solve the initial value problem

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = e^x, \quad y(0) = 1, \quad y'(0) = 0. \quad (20\%)$$